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Abstract: We study *directed bigraph with negative ports*, a bigraphical framework for representing models for distributed, concurrent and ubiquitous computing. With respect to previous versions, we add the possibility that components may *govern* the access to resources, like (web) servers control requests from clients. This framework encompasses many common computational aspects, such as name or channel creation, references, client/server connections, localities, etc, still allowing to derive systematically labelled transition systems whose bisimilarities are congruences.

As application examples, we analyse the encodings of client/server communications through firewalls, of (compositional) Petri nets and of chemical reactions.

Keywords: Bigraphs, reactive systems, (open) Petri nets, graph-based approaches to service-oriented applications.

1 Introduction

Bigraphical reactive systems (BRSs) are an emerging graphical framework proposed by Milner and others [Mil01, Mil06] as a unifying theory of process models for distributed, concurrent and ubiquitous computing. A bigraphical reactive system consists of a category of *bigraphs* (usually generated over a given *signature of controls*) and a set of *reaction rules*. Bigraphs can be seen as representations of the possible configurations of the system, and the reaction rules specify how these configuration can evolve, i.e., the reaction relation between bigraphs. Often, bigraphs represent terms up-to structural congruence and reaction rules represent term rewrite rules.

Many process calculi have successfully represented as bigraphical reactive systems: λ-calculus [Mil07], CCS [Mil06], π-calculus [BS06, JM04], Mobile Ambients [Jen08], Homer [BH06], Fusion [GM07b], Petri nets [LM06], and context-aware systems [BDE$^+$06]. The advantage of using bigraphical reactive systems is that they provide general results for deriving a labelled transition system *automatically* from the reaction rules, via the so-called *IPO construction*. Notably, the bisimulation on this transition system is always a congruence; thus, bigraphical reactive systems provide general tools for compositional reasoning about concurrent, distributed systems.

Bigraphs are the key structures supporting these results. A bigraph is a set of nodes (the *controls*), endowed with two independent structures, the *place graph* and the *link graph* (Figure 1). The place graph is a tree over the nodes, representing the spatial arrangement (i.e., nesting) of the various components of the system. The link graph represents the communication connections, possibly traversing the place structure. A bigraph may be "not ground", in the sense that it may have one or more "holes", or *sites* (the gray boxes) to be instantiated; these holes are specific leaves of the place graph, where other bigraphs can be grafted, respecting the connection links. This notion of composition between bigraphs yields a categorical structure.
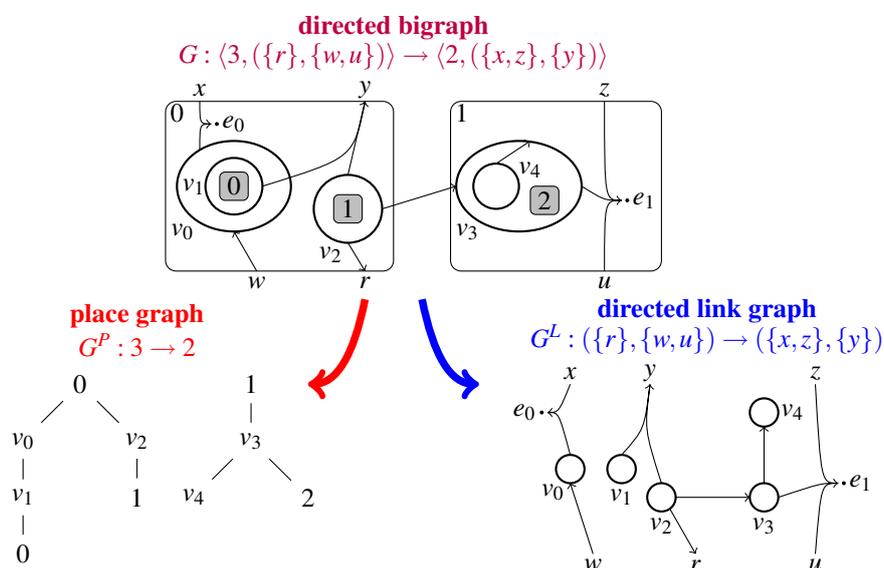
Figure 1: An example of directed bigraph, with negative ports.

In Milner's "pure bigraphs" [Mil06], connections are represented by hyper-arcs between nodes. This model has been successfully used to represent many calculi, such as CCS, and (with a small variant) λ-calculus, π-calculus. Nevertheless, other calculi, such as Fusion [PV98], seem to escape this framework. Aiming to a more expressive framework, in previous work [GM07a, GM07b], we have introduced *directed bigraphs*. Pure and directed bigraphs differ only on the link structure: in the directed variant, we distinguish "edges" from "connections". Intuitively, edges represent *(delocalized) resources*, or *knowledge tokens*, which can be *accessed* by controls. Arcs are arrows from ports of controls to edges (possibly through names on the interfaces of bigraphs); moreover, in the version considered in the present paper, we allow arcs to point to other control's ports (Figure 1). Outward ports on a control represent the capability of the control to access to (external) resources; insted, inward ports represent the capability of the control to "stop" or "govern" other node's requests. The presence of both kinds of capabilities is common in distributed scenarios, such as client/server communications, firewalls, web services etc; for instance a system may ask to access to some data, but this attempt may be blocked, checked and possibly redirected by a guarding mechanism. Moreover, controls with inward ports can represent *localized* resources, that is, resources with a position within the place hierarchy; this cannot be represented easily by edges, which do not appear in the place graph.

Notably, these extended bigraphs have relative pushout (RPO) and idem pushout (IPO) constructions, admit a notion of normal form, and a sound a complete axiomatization can be given. Therefore, these bigraphs can be conveniently used for building wide reaction systems from which we can synthesize labelled transition systems, and whose bisimilarity is a congruence.

Due to lack of space, in this paper we can only skim over these theoretical results; we prefer to focus on some interesting applications of this framework. In Section 2 we give the basic

definitions about directed bigraphs. General theoretical results (e.g., normal form, complete axiomatization, RPO construction, etc.) are presented in Sections 3–5. Then, Section 6 is devoted to example applications. We apply this framework to the representation of distributed services and protocols (taking a three-tier architecture with a firewall as an example) and of chemical reactions. Moreover, we consider the encoding of Petri nets in greater detail; in particular, we show that the IPO bisimilarity is sound with respect to the η-bisimilarity of open Petri nets.

Conclusions and direction for future work are in Section 7.

2 Directed bigraphs over polarized signatures

In this section we introduce directed bigraphs, with inward ("negative") ports on controls, extending [GM07a]. Following previous developments about pure and directed bigraphs, we work in *supported monoidal precategories*; we refer to [JM04, §3] for an introduction.

A *polarized signature* is a signature of controls, which may have two kind of ports: *negative* and *positive*. Let $\mathscr{K}$ be a polarized signature; we denote with $ar^n, ar^p : \mathscr{K} \to \mathbb{N}$ the arity functions of the negative and positive ports, respectively. Thus, for $k \in \mathscr{K}$, the arity function is $ar(k) \triangleq (ar^n(k), ar^p(k))$. A control k is *positive* if it has only positive ports (i.e., $ar^n(k) = 0$); it is *negative* if it has only negative ports (i.e., $ar^p(k) = 0$).

Definition 1 A *polarized interface X* is a pair of sets of names $X = (X^-, X^+)$; the two components are called *downward* and *upward* interfaces, respectively.

A *directed link graph $A : X \to Y$* is $A = (V, E, ctrl, link)$ where X, Y are the *inner* and *outer* interfaces, V is the set of *nodes*, E is the set of *edges*, $ctrl : V \to \mathscr{K}$ is the *control map*, and $link : \mathsf{Pnt}(A) \to \mathsf{Lnk}(A)$ is the *link map*, where *ports*, *points* and *links* of A are defined as follows:

$$\mathsf{Prt}^n(A) \triangleq \sum_{v \in V} ar^n(ctrl(v)) \quad \mathsf{Prt}^p(A) \triangleq \sum_{v \in V} ar^p(ctrl(v)) \quad \mathsf{Prt}(A) \triangleq \mathsf{Prt}^n(A) \cup \mathsf{Prt}^p(A)$$

$$\mathsf{Pnt}(A) \triangleq X^+ \uplus Y^- \uplus \mathsf{Prt}^p(A) \qquad \mathsf{Lnk}(A) \triangleq X^- \uplus Y^+ \uplus \mathsf{Prt}^n(A) \uplus E$$

The link map cannot connect downward and upward names of the same interface, i.e., the following condition must hold: $(link(X^+) \cap X^-) \cup (link(Y^-) \cap Y^+) = \emptyset$; moreover the link map cannot connect positive and negative ports of the same node.

Directed link graphs are graphically depicted much like ordinary link graphs, with the difference that edges are explicit objects, and not hyper-arcs connecting points and names; points and names are associated to links (that is edges or negative ports) or other names by (simple, non hyper) directed arcs. An example are given in Figure 1. This notation aims to make explicit the "resource request flow": positive ports and names in the interfaces can be associated either to internal or to external resources. In the first case, positive ports and names are connected to an edge or a negative port; these names are "inward" because they offer to the context the access to an internal resource. In the second case, the positive ports and names are connected to an "outward" name, which is waiting to be plugged by the context into a resource.

In the following, by "signature", "interface" and "link graphs" we will intend "polarized signature", "polarized interface" and "directed link graphs" respectively, unless otherwise noted.

Definition 2   The precategory of *directed link graphs* has polarized interfaces as objects, and directed link graphs as morphisms.

Given two directed link graphs $A_i = (V_i, E_i, ctrl_i, link_i) : X_i \rightarrow X_{i+1}$ $(i = 0, 1)$, the composition $A_1 \circ A_0 : X_0 \rightarrow X_2$ is defined when the two link graphs have disjoint nodes and edges. In this case, $A_1 \circ A_0 \triangleq (V, E, ctrl, link)$, where $V \triangleq V_0 \uplus V_1$, $ctrl \triangleq ctrl_0 \uplus ctrl_1$, $E \triangleq E_0 \uplus E_1$ and

$$link : X_0^+ \uplus X_2^- \uplus \mathsf{Prt}^p(A_0) \uplus \mathsf{Prt}^p(A_1) \rightarrow X_0^- \uplus X_2^+ \uplus E \uplus \mathsf{Prt}^n(A_0) \uplus \mathsf{Prt}^n(A_1)$$

is defined as follows:

$$link(p) \triangleq \begin{cases} link_0(p) & \text{if } p \in X_0^+ \uplus \mathsf{Prt}^p(A_0) \text{ and } link_0(p) \in X_0^- \uplus E_0 \uplus \mathsf{Prt}^n(A_0) \\ link_1(x) & \text{if } p \in X_0^+ \uplus \mathsf{Prt}^p(A_0) \text{ and } link_0(p) = x \in X_1^+ \\ link_1(p) & \text{if } p \in X_2^- \uplus \mathsf{Prt}^p(A_1) \text{ and } link_1(p) \in X_2^+ \uplus E_1 \uplus \mathsf{Prt}^n(A_1) \\ link_0(x) & \text{if } p \in X_2^- \uplus \mathsf{Prt}^p(A_1) \text{ and } link_1(p) = x \in X_1^-. \end{cases}$$

The identity link graph of X is $id_X \triangleq (\emptyset, \emptyset, \emptyset_{\mathscr{K}}, id_{X^-} \cup id_{X^+}) : X \rightarrow X$.

It is easy to check that composition is associative, and that given a link graph $A : X \rightarrow Y$, the compositions $A \circ id_X$ and $id_Y \circ A$ are defined and equal to A.

Definition 1 forbids connections between names of the same interface in order to avoid undefined link maps after compositions. Similarly, links between ports on the same node are forbidden, because these graphs cannot be obtained by composing an "unlinked" node and a context.

It is easy to see that the precategory $'$DLG is self-dual, that is $'$DLG $\cong$ $'$DLG^{op}.

The notions of openness, closeness, leanness, etc. defined in [GM07a] can be easily extended to the new framework, considering negative ports as a new kind of resources. Moreover, the definition of tensor product can be derived extending to negative ports the one given in [GM07a].

Finally, we can define the *(extended) directed bigraphs* as the composition of standard place graphs (see [JM04, §7] for definitions) and directed link graphs.

Definition 3   A *directed bigraph* with signature $\mathscr{K}$ is $G = (V, E, ctrl, prnt, link) : I \rightarrow J$, where $I = \langle m, X \rangle$ and $J = \langle n, Y \rangle$ are its inner and outer interfaces respectively. An interface is composed by a *width* (a finite ordinal) and by a pair of finite sets of names. V and E are the sets of nodes and edges respectively, and *prnt*, *ctrl* and *link* are the parent, control and link maps, such that $G^P \triangleq (V, ctrl, prnt) : m \rightarrow n$ is a place graph and $G^L \triangleq (V, E, ctrl, link) : X \rightarrow Y$ is a directed link graph.

We denote G as combination of G^P and G^L by $G = \langle G^P, G^L \rangle$. In this notation, a place graph and a (directed) link graph can be put together iff they have the same sets of nodes.

Definition 4   The precategory $'$DBIG of directed bigraph with signature $\mathscr{K}$ has interfaces $I = \langle m, X \rangle$ as objects and directed bigraphs $G = \langle G^P, G^L \rangle : I \rightarrow J$ as morphisms. If $H : J \rightarrow K$ is another directed bigraph with sets of nodes and edges disjoint from the respectively ones of G, then their composition is defined by composing their components, i.e.:

$$H \circ G \triangleq \langle H^P \circ G^P, H^L \circ G^L \rangle : I \rightarrow K.$$

The identity directed bigraph of $I = \langle m, X \rangle$ is $\langle id_m, id_X \rangle : I \rightarrow I$.

Analogously, the tensor product of two bigraphs can be defined tensoring their components.

It is easy to check that for every signature $\mathscr{K}$, the precategory $'$DBIG is wide monoidal; the origin is $\varepsilon = \langle 0, (\emptyset, \emptyset) \rangle$ and the interface $\langle n, X \rangle$ has width n. Hence, $'$DBIG can be used for applying the theory of *wide reaction systems* and *wide transition systems* as developed by Jensen and Milner; [JM04, §4, §5]. To this end, we need to show that $'$DBIG has RPOs and IPOs. Since place graphs are as usual, it suffices to show that directed link graphs have RPOs and IPOs.

Theorem 1 *If a pair $\vec{A}$ of link graphs has a bound $\vec{D}$, there exists an RPO $(\vec{B}, B)$ for $\vec{A}$ to $\vec{D}$.*

As a consequence, $'$DLG has IPOs too. See section 4 for the direct construction for RPOs in directed bigraphs with negative ports, extending the construction given in [GM07a].

Actually, often we do not want to distinguish bigraphs differing only on the identity of nodes and edges. To this end, we introduce the category DBIG of *abstract directed bigraphs*, which is constructed from $'$DBIG forgetting the identity of nodes and edges and any idle edge. More precisely, abstract bigraphs are bigraphs taken up-to an equivalence $\rightleftharpoons$ (see [JM04] for details).

Definition 5 Two concrete directed bigraphs G and H are *lean-support equivalent*, written $G \rightleftharpoons H$, if there exists an iso between their nodes and edges sets after removing any idle edges.

The category DBIG of abstract directed bigraphs has the same objects as $'$DBIG, and its arrows are lean-support equivalence classes of directed bigraphs.

3 Algebra and Axiomatization

In this section, generalizing similar results for directed bigraphs, we describe the main classes of bigraphs and the elementary bigraphs which can generate all bigraphs according to a well-defined normal form. As main result we give a normalization theorem. We refer the reader to [GM08] for a detailed presentation of the notation used here.

First, we introduce two distinct and complementary subclasses of bigraphs: *wirings* and *discrete bigraphs*, that are used in defining the normal form and the axiomatization.

Definition 6 A *wiring* is a bigraph whose interfaces have zero width (and hence has no nodes). The wirings ω are generated by the composition or tensor product of three elements: substitutions $\sigma : (\emptyset, X^+) \to (\emptyset, Y^+)$, fusions $\delta : (Y^-, \emptyset) \to (X^-, \emptyset)$, and closures ${\textstyle\bigwedge}_y^x : (\emptyset, y) \to (x, \emptyset)$.

Definition 7 An interface is *prime* if its width is 1. Often we abbreviate a prime interface $\langle 1, (X^-, X^+) \rangle$ with $\langle (X^-, X^+) \rangle$, in particular $1 = \langle (\emptyset, \emptyset) \rangle$. A prime bigraph $P : \langle m, (Y^-, Y^+) \rangle \to \langle (X^-, X^+) \rangle$ has a prime outer interface and the names in Y^+, X^- are linked to negative ports of P.

An important prime bigraph is $merge_m : m \to 1$, it has no nodes and maps m sites to one root.

Definition 8 A bigraph is *discrete* if it has no edges and every open link has exactly one point.

The discreteness is well-behaved, and preserved by composition and tensor. It is easy to see that discrete bigraphs form a monoidal sub-precategory of $'$DBIG.

$$\mathbf{Y}_y^x : (\emptyset, y) \to (x, \emptyset)$$
closure

$$\triangle_X^y : (\emptyset, X) \to (\emptyset, y)$$
substitution

$$\nabla_x^Y : (x, \emptyset) \to (Y, \emptyset)$$
fusion

$$K(l) : \langle (\vec{x}^-, \vec{Y}^+) \rangle \to \langle (\vec{Y}^-, \vec{x}^+) \rangle$$
a discrete ion

$$1 : \varepsilon \to 1$$
a barren root

$$merge : 2 \to 1$$
joining 2 sites in 1 root

$$\gamma_{1,1} : 2 \to 2$$
swapping 2 sites
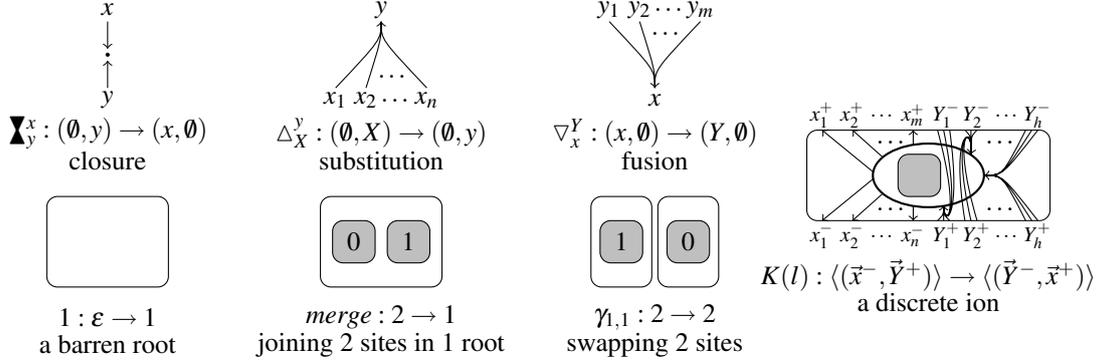
Figure 2: Elementary Bigraphs over polarized signatures.

Definition 9 Let K be any non atomic control with arity (k^-, k^+), let $\vec{x}^-, \vec{x}^+$ be two sequences of distinct names, and let $\vec{Y}^+, \vec{Y}^-$ be two sequences of (possibly empty) sets of distinct names, such that: $|\vec{x}^-| + |\vec{x}^+| = k^+$ and $|\vec{Y}^-| = |\vec{Y}^+| = k^-$. For a K-node v, we define the discrete *ion*

$$K(v,l) : \langle (\vec{x}^-, \vec{Y}^+) \rangle \to \langle (\vec{Y}^-, \vec{x}^+) \rangle$$

as the bigraph with exactly a node v and l is a pair of maps: an iso map $l^p : \vec{x}^- \cup \vec{x}^+ \to \mathsf{Prt}^p(v)$ describing the linking among positive ports and names in $\vec{x}^-$ or $\vec{x}^+$, and another iso map $l^n : \vec{Y}^- \cup \vec{Y}^+ \to \mathsf{Prt}^n(v)$ describing the linking among negative ports and sets of upward inner names (in $\vec{Y}^+$) and sets of downward outer names (in $\vec{Y}^-$). We omit v when it can be understood.

For a prime discrete bigraph P with outer names in (Z^-, Z^+), we define a discrete *molecule* as:

$$(K(l) \otimes id_{(Z^- \backslash \vec{x}^-, Z^+ \backslash \vec{Y}^+)}) \circ P.$$

If K is atomic, we define the discrete *atom*, as an ion without the site

$$K(l) : (\vec{x}^-, \vec{Y}^+) \to \langle (\vec{Y}^-, \vec{x}^+) \rangle.$$

An arbitrary (non-discrete) ion, molecule or atom is formed by the composition of $\omega \otimes id_1$ with a discrete one. Often we omit $\ldots \otimes id_I$ in the compositions, when there is no ambiguity.

Figure 2 shows the *algebraic signature*, that is a set of elementary bigraphs which allow to define any other bigraph using solely composition and tensor product. Before giving a formal result, we provide an intuitive explanation of the meaning of these elementary bigraphs.

- The first three bigraphs build up all wirings, i.e. all link graphs having no nodes. All substitutions (fusions, resp.) can be obtained tensoring elementary substitutions $\triangle_X^y$ (fusions ∇_x^Y, resp.); the tensor products of singleton substitutions $\triangle_x^y$ or singleton fusions ∇_y^x give all renamings. Composition and tensor product of substitutions, fusions and closures give all wirings.

- The next three bigraphs define all placings, i.e. all place graphs having no nodes; for example $merge_m : m \to 1$, merging m sites in a unique root, are defined as:

$$merge_0 \triangleq 1 \qquad merge_{m+1} \triangleq merge \circ (id_1 \otimes merge_m).$$

Notice that $merge_1 = id$ and $merge_2 = merge$, and that all permutations π are constructed by composition and tensor product from the place symmetry $\gamma_{1,1}$.

- Finally, for expressing any direct bigraph we need to introduce only the discrete ions $K(l)$: $\langle(\vec{x}^-, \vec{Y}^+)\rangle \to \langle(\vec{Y}^-, \vec{x}^+)\rangle$. In particular, we can express any discrete atoms as $K(l) \circ 1$.

The following proposition shows that every bigraph can be expressed in a normal form, called *discrete diagonal normal form* (DDNF). We will use D, Q and N to denote discrete, prime and discrete bigraphs, and discrete molecules respectively.

Theorem 2 *1. A bigraph G on signature $\mathcal{K}$ can be expressed uniquely (up to iso) as:*

$$G = (\omega \otimes id_n) \circ D \circ (\omega' \otimes id_m) \tag{1}$$

where D is a discrete bigraph and ω, ω' are two wirings satisfying the following conditions:

  i. in ω, if two outer downward names are peer, then their target is an edge;

  ii. in ω' there are no edges, and no two inner upward names are peer.

2. Every discrete bigraph $D : \langle n, Z\rangle \to \langle m, W\rangle$, may be factored uniquely (up to iso) on the width m in the following form:

$$D = \alpha \otimes \left(\begin{pmatrix} (Q_0 \otimes \chi_{1,0}^{out} \otimes \chi_{2,0}^{out} \otimes \ldots &)\circ \\ (\chi_{0,1}^{in} \otimes Q_1 \otimes \chi_{2,1}^{out} \otimes \ldots &)\circ \\ (\chi_{0,2}^{in} \otimes \chi_{1,2}^{in} \otimes Q_2 \otimes \ldots &)\circ \\ & \ddots \\ (& \ldots \otimes Q_{m-3} \otimes \chi_{m-2,m-3}^{out} \otimes \chi_{m-1,m-3}^{out})\circ \\ (& \ldots \otimes \chi_{m-3,m-2}^{in} \otimes Q_{m-2} \otimes \chi_{m-1,m-2}^{out})\circ \\ (& \ldots \otimes \chi_{m-3,m-1}^{in} \otimes \chi_{m-2,m-1}^{in} \otimes Q_{m-1}) \end{pmatrix} \circ (\pi \otimes id_{dom(\vec{Q})}) \right)$$

shortly:

$$D = \alpha \otimes (\chi(Q_0, \ldots, Q_{m-1}) \circ (\pi \otimes id_{dom(\vec{Q})})) \tag{2}$$

where α is a renaming, π is a permutation, each $Q_i : \langle h_i, (X_i^-, X_i^+)\rangle \to \langle(Y_i^-, Y_i^+)\rangle$ is prime and discrete, and $\chi_{i,l}^{out}$ ($\chi_{i,l}^{in}$ resp.) are the identities on the outer (inner resp.) names of Q_i at level $l \in \{0, \ldots, m-1\}$, which are not used by $Q_l, \ldots, Q_{i-1}$ ($Q_{i+1}, \ldots, Q_l$ resp.). Formally:

$$\chi_{i,l}^{out} \triangleq id_{\langle(Y_i^- \setminus (\uplus_{j=l}^{i-1} X_j^-), Y_i^+ \setminus (\uplus_{j=l}^{i-1} X_j^+))\rangle} \qquad \text{for } i > l$$

$$\chi_{i,l}^{in} \triangleq id_{\langle h_i, (X_i^- \setminus (\uplus_{j=i+1}^{l} Y_j^-), X_i^+ \setminus (\uplus_{j=i+1}^{l} Y_j^+))\rangle} \qquad \text{for } i < l.$$

3. Every prime and discrete Q, may be factored uniquely (up to iso) in the following form:

$$Q = (merge_{n+p} \otimes id_{codom(\vec{N})}) \circ (id_n \otimes \chi(N_0, \ldots, N_{p-1})) \circ (\pi \otimes id_{dom(\vec{N})}) \tag{3}$$

where χ is defined as in 2.

4. *Every discrete molecule N, may be factored uniquely (up to iso) in the following form:*

$$N = (K(l) \otimes id_{(codom(Q) \setminus (\vec{x}^-, \vec{Y}^+))}) \circ Q. \tag{4}$$

Furthermore, the expression is unique up to iso on the parts and reordering of Ns in Q.

These equations can be used for normalizing any bigraph G as follows; first, we apply equations (1), (2) to G once, obtaining an expression containing prime discrete bigraphs $Q_0, \ldots, Q_{m-1}$. These are decomposed further using equations (3), (4) repeatedly: each Q_i is decomposed into an expression containing molecules $N_{i,0}, \ldots, N_{i,p_i-1}$, each of which is decomposed in turn into an ion containing another prime discrete $Q'_{i,j}$. The last two steps are repeated recursively until the ions are atoms or have only holes as children. Note that 1 is a special case of Q when $n = p = 0$.

Notice that if the signature has only positive controls, then all the "Q_i layers" can be collapsed in a unique one, obtaining a normal form that is very similar to the Milner's DNF (see [Mil06]).

Furthermore, a renaming is discrete but not prime (since it has zero width); this is why the factorization has such a factor. The uniqueness of that factorization depends on the fact that prime bigraphs have upward inner names and downward outer names linked to negative ports.

In Table 1 we give a set of axioms which we prove to be sound and complete.

Each of these equations holds only when both sides are defined; in particular, recall that the tensor product of two bigraphs is defined only if the name sets are disjoint. It is important to notice that for ions only the renaming axiom is needed (because names are treated positionally).

Theorem 3 *Let E_0, E_1 be two expressions constructed from the elementary bigraphs by composition and tensor product. Then, E_0 and E_1 denote the same bigraph in DBIG if and only if the equation $E_0 = E_1$ can be proved by the axioms in Table 1.*

Sharing products To conclude this section ,we define some variants of the tensor product, which allow the sharing of names. Actually, these operators can be defined in terms of composition and tensor, but they turn out quite useful in the encoding of process calculi, where in the usual "parallel" $P \mid Q$, processes P and Q can share names. In directed bigraphs, this sharing can involve inner downward names and/or outer upward names, as described by the following definitions.

Definition 10 The *outer sharing product* ($\curlywedge$), *inner sharing product* ($\curlyvee$) and *sharing product* ($\parallel$) of two objects $X = (X^-, X^+), Y = (Y^-, Y^+)$ and of two link graphs $A_i : X_i \to Y_i$ $(i = 0, 1)$ are defined as follows:

$$(X^-, X^+) \curlywedge (Y^-, Y^+) \triangleq (X^- \uplus Y^-, X^+ \cup Y^+)$$
$$(X^-, X^+) \curlyvee (Y^-, Y^+) \triangleq (X^- \cup Y^-, X^+ \uplus Y^+)$$
$$A_0 \curlywedge A_1 \triangleq (V_0 \uplus V_1, E_0 \uplus E_1, ctrl_0 \uplus ctrl_1, link_0 \uplus link_1) : X_0 \otimes X_1 \to Y_0 \curlywedge Y_1$$
$$A_0 \curlyvee A_1 \triangleq (V_0 \uplus V_1, E_0 \uplus E_1, ctrl_0 \uplus ctrl_1, link_0 \uplus link_1) : X_0 \curlyvee X_1 \to Y_0 \otimes Y_1$$
$$A_0 \parallel A_1 \triangleq (V_0 \uplus V_1, E_0 \uplus E_1, ctrl_0 \uplus ctrl_1, link_0 \uplus link_1) : X_0 \curlyvee X_1 \to Y_0 \curlywedge Y_1$$

defined when interfaces are defined and A_i have disjoint node and edge sets.

Categorical Axioms

$$A \circ id = A = id \circ A \qquad A \circ (B \circ C) = (A \circ B) \circ C$$

$$A \otimes id_\varepsilon = A = id_\varepsilon \otimes A \qquad A \otimes (B \otimes C) = (A \otimes B) \otimes C$$

$$\gamma_{I,\varepsilon} = id_I \qquad \gamma_{J,I} \circ \gamma_{I,J} = id_{I \otimes J} \qquad \gamma_{I \otimes J, K} = (\gamma_{I,K} \otimes id_J) \circ (id_I \otimes \gamma_{J,K})$$

$$(A_1 \otimes B_1) \circ (A_0 \otimes B_0) = (A_1 \circ A_0) \otimes (B_1 \circ B_0)$$

$$\gamma_{I,K} \circ (A \otimes B) = (B \otimes A) \circ \gamma_{H,J} \qquad (\text{where } A : H \to I, B : J \to K)$$

Link Axioms

$$\triangle_x^x = id_{(\emptyset,x)} \quad \nabla_x^x = id_{(x,\emptyset)} \quad \mathbf{Y}_y^x \circ \triangle_z^y = \mathbf{Y}_z^x \quad \nabla_x^z \circ \mathbf{Y}_y^x = \mathbf{Y}_y^z \quad \nabla_x \circ \mathbf{Y}_y^x \circ \triangle^y = id_\varepsilon$$

$$\triangle_{(Y \uplus y)}^z \circ (id_{(\emptyset,Y)} \otimes \triangle_X^y) = \triangle_{(Y \uplus X)}^z \qquad (id_{(Y,\emptyset)} \otimes \nabla_y^X) \circ \nabla_z^{(Y \uplus y)} = \nabla_z^{(X \uplus Y)}$$

Place Axioms

$$merge \circ (1 \otimes id_1) = id_1 \qquad merge \circ \gamma_{1,1} = merge$$

$$merge \circ (merge \otimes id_1) = merge \circ (id_1 \otimes merge)$$

Node Axioms

Let $K(l) : \langle (\vec{x}^-, \vec{Y}^+) \rangle \to \langle (\vec{Y}^-, \vec{x}^+) \rangle$

$$(id_1 \otimes \alpha^+ \otimes \zeta^-) \circ K(l) = K(l') \text{ with } l' = ((id_{\vec{x}^-} \cup \alpha^+) \circ l^p, ((\zeta^-)^{op} \cup id_{\vec{Y}^+}) \circ l^n)$$

$$K(l) \circ (id_1 \otimes \alpha^- \otimes \zeta^+) = K(l'') \text{ with } l'' = ((\alpha^- \cup id_{\vec{x}^+}) \circ l^p, (id_{\vec{Y}^-} \cup (\zeta^+)^{op}) \circ l^n)$$

where $\alpha^+ : \vec{x}^+ \to \vec{z}^+, \alpha^- : \vec{x}^- \to \vec{z}^-$ are renamings and

$\zeta^- : \vec{Z}^- \to \vec{Y}^-, \zeta^+ : \vec{Z}^+ \to \vec{Y}^+$ are surjective fusions.

Table 1: Axiomatization for the abstract directed bigraphs.

The *outer sharing product*, *inner sharing product* and *sharing product* of two objects $I = \langle m, X \rangle, J = \langle n, Y \rangle$ and of two bigraphs $G_i : I_i \to J_i$ $(i = 0, 1)$ are defined by extending the corresponding products on their link graphs with the tensor product on widths and place graphs:

$$\langle m, X \rangle \curlywedge \langle n, Y \rangle \triangleq \langle n + m, X \curlywedge Y \rangle \qquad \langle m, X \rangle \curlyvee \langle n, Y \rangle \triangleq \langle n + m, X \curlyvee Y \rangle$$

$$G_0 \curlywedge G_1 \triangleq \langle G_0^P \otimes G_1^P, G_0^L \curlywedge G_1^L \rangle : I_0 \otimes I_1 \to J_0 \curlywedge J_1$$

$$G_0 \curlyvee G_1 \triangleq \langle G_0^P \otimes G_1^P, G_0^L \curlyvee G_1^L \rangle : I_0 \curlyvee I_1 \to J_0 \otimes J_1$$

$$G_0 \parallel G_1 \triangleq \langle G_0^P \otimes G_1^P, G_0^L \parallel G_1^L \rangle : I_0 \curlyvee I_1 \to J_0 \curlywedge J_1.$$

defined when interfaces are defined and G_i have disjoint node and edge sets.

It is simple to verify that $\curlywedge$, $\curlyvee$ and $\parallel$ are associative, with unit ε.

4 Construction of RPOs

We first give an idea of how the construction works. Suppose $D_0 : X_0 \to Z$, $D_1 : X_1 \to Z$ is a bound for a span $A_0 : W \to X_0$, $A_1 : W \to X_1$ and we wish to construct the RPO $(B_0 : X_0 \to \hat{X}, B_1 : X_1 \to \hat{X}, B : \hat{X} \to Z)$. In the following we will denote a pair (A_0, A_1) by $\vec{A}$ and the link map of A simply by A. To form the pair $\vec{B}$ we truncate $\vec{D}$ by removing all the edges, nodes and ports not present in $\vec{A}$. Then in the outer interface of $\vec{B}$, we create an outer name for each point unlinked by the truncation: the downward names connected to the same link (name, edge or negative port) must be "bound together", i.e. we must consider all the possible ways to associate a downward name of A_0 with one of A_1 and vice versa; further we must equate an upward name of A_0 with one of A_1 if they are both connected to a point shared between A_0 and A_1. Formally:

Construction 1   A relative pushout $(\vec{B} : \vec{X} \to \hat{X}, B : \hat{X} \to Z)$, for a pair $\vec{A} : W \to \vec{X}$ of link graphs relative to a bound $\vec{D} : \vec{X} \to Z$, will be built in three stages. Since RPOs are preserved by isomorphisms, we can assume the components of X_0 and X_1 disjoint.

nodes and edges   If V_i are the nodes of A_i $(i = 0, 1)$, then the nodes of D_i are $V_{D_i} = (V_{\bar{i}} \setminus V_i) \uplus V_2$ for some V_2. Define the nodes of B_i and B to be $V_{B_i} \triangleq V_{\bar{i}} \setminus V_i$ $(i = 0, 1)$ and $V_B \triangleq V_2$. Edges E_i and ports P_i^p, P_i^n of A_i are treated analogously.

interface   Construct the shared codomain $\hat{X} = (\hat{X}^-, \hat{X}^+)$ of $\vec{B}$ as follows: first we define the names in each $X_i = (X_i^-, X_i^+)$, for $i = 0, 1$, that must be mapped into $\hat{X} = (\hat{X}^-, \hat{X}^+)$:

$$X_i'^- \triangleq \{x \in X_i^- \mid \exists y \in X_{\bar{i}}^- \text{ s.t. } A_i(x) = A_{\bar{i}}(y) \text{ or } A_i(x) \in (E_i \setminus E_{\bar{i}}) \uplus (P_i^n \setminus P_{\bar{i}}^n)\}$$
$$X_i'^+ \triangleq \{x \in X_i^+ \mid D_i(x) \in (E_2 \uplus P_2^n \uplus Z^+)\}.$$

Let $\text{Res} \triangleq W^- \uplus (E_0 \cap E_1) \uplus (P_0^n \cap P_1^n)$. We define for each $l \in \text{Res}$ the set of names in $X_i'^-$ linked to l:

$$X_i'^-(l) \triangleq \{x \in X_i'^- \mid A_i(x) = l\} \quad (i = 0, 1).$$

Now we must "bind together" names connected to the same link, so we create all the possible pairs between a name in $X_0'^-$ and a name in $X_1'^-$. Further we must add to $\hat{X}^-$ all the names in $X_i'^-$ "not associable" to any name of $X_{\bar{i}}'^-$. Then the set of downward names of $\vec{B}$ is:

$$\hat{X}^- \triangleq \bigcup_{l \in \text{Res}} X_0'^-(l) \times X_1'^-(l) \cup \sum_{i \in \{0,1\}} \bigcup_{e \in (E_i \setminus E_{\bar{i}}) \uplus (P_i^n \setminus P_{\bar{i}}^n)} X_i'^-(e).$$

Next, on the disjoint sum $X_0'^+ + X_1'^+$, define $\cong$ to be the smallest equivalence for which $(0, x_0) \cong (1, x_1)$ iff there exists $p \in W^+ \uplus (P_0^p \cap P_1^p)$ such that $A_0(p) = x_0$ and $A_1(p) = x_1$. Then define:

$$\hat{X}^+ \triangleq (X_0'^+ + X_1'^+)/_\cong.$$

For each $x \in X_i'^+$ we denote the equivalence class of (i, x) by $\widehat{i, x}$.
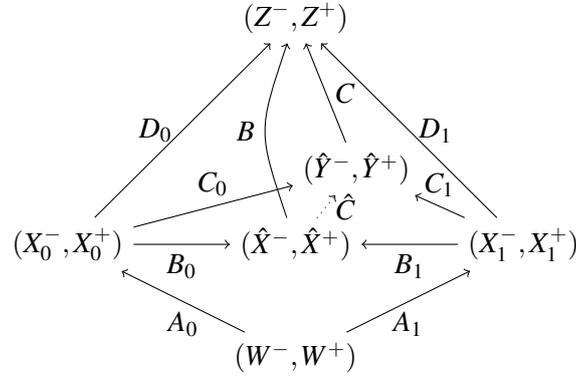
Figure 3: Construction of the unique arrow form the RPO to any other candidate.

links Define the link maps of B_i as follows:

$$\text{for } x \in X_i^+ : \qquad B_i(x) \triangleq \begin{cases} D_i(x) & \text{if } x \in (X_i^+ \setminus X_i'^+) \\ \widehat{i,x} & \text{if } x \in X_i'^+; \end{cases}$$

$$\text{for } p \in P_i^p \setminus P_i^p : \qquad B_i(p) \triangleq \begin{cases} D_i(p) & \text{if } A_{\bar{i}}(p) \notin X_i^+ \\ \widehat{i,x} & \text{if } A_{\bar{i}}(p) = x \in X_{\bar{i}}^+; \end{cases}$$

$$\text{for } \hat{x} \in \hat{X}^- : \qquad B_i(\hat{x}) \triangleq \begin{cases} x & \text{if } \hat{x} = (x,y) \text{ and } i = 0 \\ y & \text{if } \hat{x} = (x,y) \text{ and } i = 1 \\ \hat{x} & \text{if } \hat{x} \in (\hat{X}^- \cap X_i^-) \\ A_{\bar{i}}(\hat{x}) & \text{if } \hat{x} \in (\hat{X}^- \cap X_{\bar{i}}^-). \end{cases}$$

Finally we define the link map of B:

$$\text{for } \hat{x} \in \hat{X}^+ : \quad B(\hat{x}) \triangleq D_i(x) \text{ where } \hat{x} = \widehat{i,x} \text{ and } x \in X_i^+;$$

$$\text{for } p \in P_2^p \uplus Z^- : \quad B(p) \triangleq \begin{cases} D_i(p) & \text{if } D_i(p) \in (E_2 \uplus P_2^n \uplus Z^+) \\ D_{\bar{i}}(p) & \text{if } D_i(p) \in (E_{\bar{i}} \setminus E_i) \uplus (P_{\bar{i}}^n \setminus P_i^n) \\ D_i(p) & \text{if } D_{\bar{i}}(p) \in (E_i \setminus E_{\bar{i}}) \uplus (P_i^n \setminus P_{\bar{i}}^n) \\ (x,y) & \text{if } D_0(p) = x \in X_0^- \text{ and } D_1(p) = y \in X_1^-. \end{cases}$$

Theorem 4 *In $'$DLG, whenever a pair $\vec{A}$ of link graphs has a bound $\vec{D}$, there exists an RPO $(\vec{B}, B)$ for $\vec{A}$ to $\vec{D}$, and Construction 1 yields such an RPO.*

Proof. The proof is in two parts. First we have to check that $(\vec{B}, B)$ is an RPO candidate; this is done by long and tedious calculations. Next, for any other candidate $(\vec{C}, C)$, we have to construct the unique arrow $\hat{C}$ such that the diagram (in Figure 3) commutes. This link graph $\hat{C}$ can be

constructed as follows: let be V_C the nodes of C, for $i = 0, 1$ the set of nodes of C_i is $V_{C_i} \triangleq (V_{\bar{i}} \setminus V_i) \uplus V_3$, where V_3 is such that $V_2 = V_3 \uplus V_C$; edges E_{C_i} and ports P_{C_i} of C_i are defined analogously. Then $\hat{C}$ has V_3, E_3 and P_3 as sets of nodes, edges and ports respectively. Its link map is defined as follows:

$$\text{for } \widehat{j,x} \in \hat{X}^+ : \qquad \hat{C}(\widehat{j,x}) \triangleq C_j(x);$$

$$\text{for } p \in P_3^p \uplus \hat{Y}^- : \qquad \hat{C}(p) \triangleq \begin{cases} C_i(p) & \text{if } C_i(p) \in (E_3 \uplus P_3^n \uplus \hat{Y}^+) \\ C_0(p) & \text{if } C_0(p) \in (\hat{X}^- \cap X_0^-) \\ C_1(p) & \text{if } C_1(p) \in (\hat{X}^- \cap X_1^-) \\ (x, y) & \text{if } C_0(p) = x \in X_0^- \text{ and } C_1(p) = x \in X_1^- \end{cases}$$

$\square$

As an immediate consequence, since $'$DLG is self-dual, we can calculate RPBs as well.

As regards IPOs, their existence is guaranteed by Theorem 4. Due to lack of space, we omit here a direct construction of IPOs, which can be obtained by adapting the one presented in [GM07a] to the RPO construction given above.

5 Directed Bigraphical Reactive and Transition Systems

In this section we introduce reactive systems and transition systems over directed bigraphs over polarized signatures. For sake of simplicity, we will keep using the names *directed bigraphical reactive systems* and *directed bigraphical transition systems* respectively, as in [GM07b]. For the definitions about wide reactive systems over monoidal precategories, see [Mil06].

Definition 11   A ground reaction rule is a pair (r, r'), where r and r' are ground with the same outer interface. Given a set of ground rules, the reaction relation $\longrightarrow$ over agents is the least, such that $Dr \longrightarrow Dr'$ for each active context D and each ground rule (r, r').

A parametric reaction rule has a redex R and reactum R', and takes the form:

$$(R : I \to J, R' : I \to J)$$

For any X and discrete $d : I \otimes X$, the parametric rule generates the ground rule:

$$((R \otimes id_X) \circ d, (R' \otimes id_X) \circ d).$$

Definition 12   A *directed bigraphical reactive system* (DBRS) over the polarized signature $\mathscr{K}$, denoted by $\mathscr{D}(\mathscr{K}, \mathscr{R})$ is the precategory $'$DBIG$(\mathscr{K})$ equipped with a set $\mathscr{R}$ of reaction rules.

Now we can prove the following important result:

Proposition 1   *DBRSs are wide reactive systems over a wide monoidal precategory*

This result ensures that DBRSs inherit from the theory of WRSs the definition of transition system based on IPOs [Mil06].

Definition 13 A *transition* for a DBRS $\mathscr{D}(\mathscr{K},\mathscr{R})$ is a quadruple (a,L,λ,a'), written as $a \xrightarrow{\;L\;}_\lambda a'$, where a and a' are ground bigraphs and there exist a ground reaction rule $(r,r') \in \mathscr{R}$ and an active context D such that $La = Dr$, and $\lambda = width(D)(width(cod(r)))$ and $a' \simeq Dr'$.

A transition is *IPO* if the (L,D) is an IPO for (a,r).

Definition 14 A *directed bigraphical transition system* (DBTS) $\mathscr{L}$ for $\mathscr{D}$ is a pair $(\mathscr{I},\mathscr{T})$, where $\mathscr{I}$ is a set of interfaces; the *agents* of $\mathscr{L}$ are the ground bigraphs with outer interfaces in $\mathscr{I}$; $\mathscr{T}$ is a set of transitions whose sources and targets are agents of $\mathscr{L}$.

The *full* (*IPO*, resp.) transition system has all interfaces, with all (IPO, resp.) transitions.

The following result follows from [Mil06, Theorem 4.6] and Proposition 1 above.

Theorem 5 *In any DBRS equipped with the IPO transition system, wide bisimilarity is a congruence.*

6 Applications

6.1 Three-tier interaction with access control

As mentioned before, directed bigraphs over polarized signatures allow to represent resource access control, by means of negative ports. This is particularly useful for representing access policies between systems, possibly in different locations; the edges can represent access tokens (or keys), which are global (although known possibly to only some controls). An example and quite common scenario is a client-server connection, where the access to the server is subject to authentication; after the request has been accepted, the server can route it to a back-end service (e.g., a DBMS); see Figure 4. The security policy is implemented by the firewall control, which allows a query to reach the server only if the client knows the correct key (rule AUTH). The server routes the query to the correct back-end service using rules like ROUTE; finally the back-end service provides the data (rule GET). An example computation is shown in Figure 5.

6.2 Compositional Petri Nets

In this section we recall briefly what a Petri net is and we give an encoding of these nets as directed bigraphs; to this end it is preferable to work with sorted links, as in [LM06]. Notice that this encoding yields naturally a notion of composition between Petri nets.

Definition 15 A *place transition net* (*P/T net*) is a 5-tuple (P,T,F,M_i) $(P \cap T = \emptyset)$, where:

- P is the set of *places; T* is the set of *transitions;*

- F is the multiset of *arcs,* linking places to transitions and vice versa: $F \triangleq \langle (P \times T) \cup (T \times P), f : (P \times T) \cup (T \times P) \to \mathbb{N} \rangle$, with the constrain $\forall t \in T.\ \exists p,q \in P.\ (p,t) \in F \wedge (t,q) \in F$;

- $M : P \to \mathbb{N}$ is a *marking,* giving to each place a number of tokens, a place p is *marked* by M if $M(p) > 0$ and *unmarked* if $M(p) = 0$; M_i is the *initial marking.*
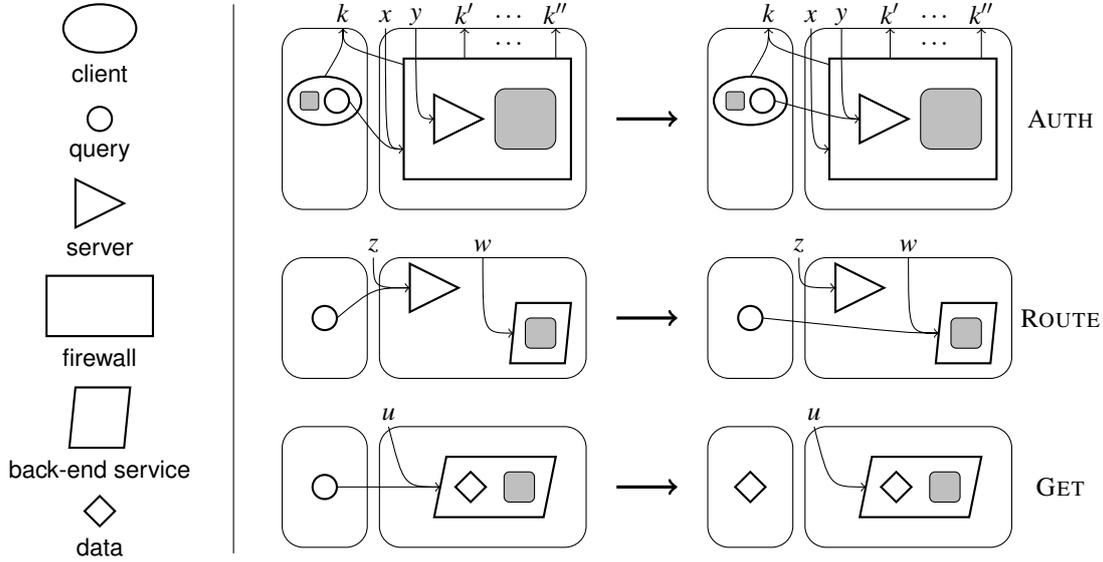
Figure 4: Signatures and rules for three-tier architecture services through a firewall.
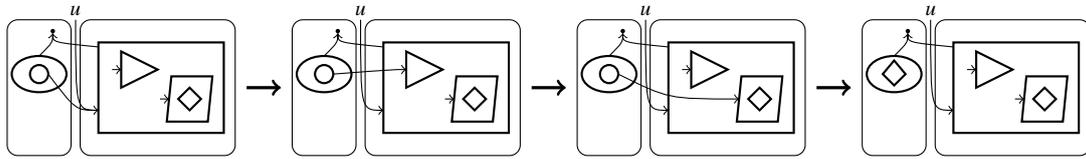
Figure 5: An example of client-server interaction through a firewall.

We define $\bullet t \triangleq \{p \mid (p,t) \in F\}$ to be the *pre-multiset* of the transition t, and $t^\bullet \triangleq \{p \mid (t,p) \in F\}$ the *post-multiset* of the transition t.

A transition t is *enabled* by a marking M if M marks every place in $\bullet t$; a transition *fires* from a marking M to a marking M', written $M \xrightarrow{t} M'$, iff for all $p \in P : M'(p) = M(p) - \sharp(\bullet p) + \sharp(p^\bullet)$, where $\sharp(\bullet p)$ and $\sharp(p^\bullet)$ are the number of occurrences of p in $\bullet t$, $t^\bullet$, respectively.

Notice that we allow multiple connections between a place and a transition, which is analogous to assign a weight to an arc representing the number of tokens to consume for firing the reaction.

Definition 16 Let $N = (P,T,F,M)$ and $N' = (P',T',F',M')$ be two P/T nets, we say that N and N' are *isomorphic*, if there exist two bijections $\alpha : P \to P'$ and $\beta : T \to T'$, such that:

- $(p,t) \in F$ iff $(\alpha(p), \beta(t)) \in F'$;

- $(t,p) \in F$ iff $(\beta(t), \alpha(p)) \in F'$;

- $M = M' \circ \alpha$.

We recall, as defined in [LM06], the definition of link sorting.
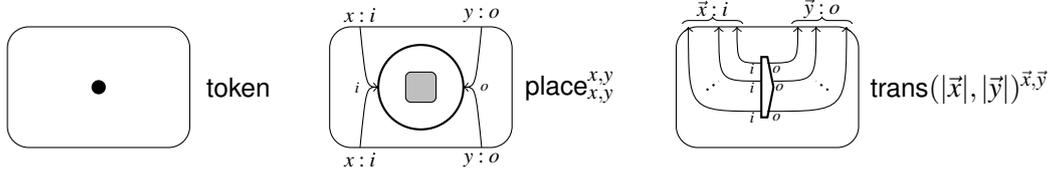
Figure 6: Signature for the encoding of compositional Petri nets.

Definition 17 A *link sorting* is a triple $\Sigma = (\Theta, \mathcal{K}, \Phi)$, where Φ is a set of sorts, and $\mathcal{K}$ is a sorted signature (that is, a signature enriched with a sort to ports of each control). Furthermore, each name in the interface (X^-, X^+) is given a sort, so the interfaces take the form $(\{x_1^- : \theta_1^-, \ldots, x_n^- : \theta_n^-\}, \{x_1^+ : \theta_1^+, \ldots, x_m^+ : \theta_m^+\})$. Finally, Φ is a rule on such enriched bigraphs, that is preserved by identities, composition and tensor product.

We denote the precategory and category of, respectively, concrete and abstract Σ-sorted directed bigraphs with $'\mathrm{DBIG}(\Sigma)$ and $\mathrm{DBIG}(\Sigma)$.

Definition 18 A *positive-negative sorting* $\Sigma = (\Theta, \mathcal{K}, \Phi)$ has sorts: $\Theta = \{\theta_1, \ldots, \theta_n\}$. The signature $\mathcal{K}$ assigns sorts to ports arbitrarily. The unique Φ rule is: a point and a link (except of edges) can be connected if they are equally sorted.

In order to define an encoding for compositional Petri nets, we introduce a positive-negative sorting Σ_{petri}, having sort $\Theta_{petri} \triangleq \{i, o\}$ and sorted signature:

$$\mathcal{K}_{petri} \triangleq \{\mathsf{token} : (0,0), \mathsf{place} : (\{1:i, 1:o\}, 0), \mathsf{trans}(h,k) : (0, \{h:i, k:o\})\} \qquad \text{where } h, k > 0$$

where the controls token and trans are both atomic, while the control place is passive (i.e., no reactions can take place inside such a node). Finally, the Φ rule ensures that the linking is allowed only among ports having the same sort. An example of use of this sorted signature is shown in Figure 6. The encoding function $[\![\cdot]\!]$ is defined as follows:

$$[\![(P,T,F,M)]\!] = merge_{(|P|+|T|)} \circ \left(id_{|P|} \curlyvee \nabla_{P \times \{i,o\}} \curlyvee \left(\bigcurlyvee_{t \in T} trans(|{}^\bullet t|, |t^\bullet|)_{({}^\bullet t \times \{i\}, \, t^\bullet \times \{o\})} \right) \right) \circ$$
$$\left(\sum_{p \in P} place_{(p,i), \, (p,o)}^{(p,i), \, (p,o)} \circ (merge_{(|M(p)|+1)} \circ (\triangle^{\{(p,i), \, (p,o)\}} \otimes (\sum_{i=0}^{M(p)} token) \otimes 1)) \right). \tag{5}$$

where, with an abuse of notation, $trans(|{}^\bullet t|, |t^\bullet|)_{({}^\bullet t \times \{i\}, \, t^\bullet \times \{o\})}$ means that if in the multisets there are some repetitions of places then the ports of $trans$ are linked to the same downward inner name (i.e., (p,i) or (p,o)), an alternative definition is to link every port of $trans$ to a different downward inner name and then (eventually) "equate" these names using fusions.

Proposition 2 *Let N, N' be two P/T nets, N is isomorphic to N' iff $[\![N]\!] = [\![N']\!]$.*

We have a different reaction rule for any pair (h,k) associated to the control trans, in Figure 7 we show the reaction rule for the pair $(3,2)$, that is a transition having 3 inputs and 2 outputs.
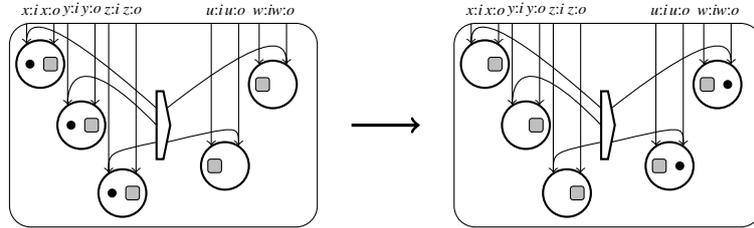
Figure 7: Example of reaction rule in the case of 3 input and 2 output places.
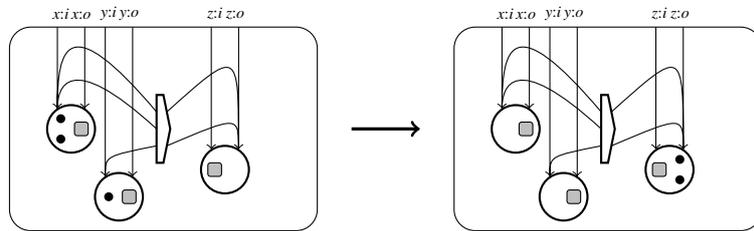
Figure 8: Example of reaction rule in the case of 2 input and 1 output places (with multiple arcs).

Moreover, we allow multiple connections between places and transitions, as in Figure 8, and we can have transitions using some places as inputs and outputs, see Figure 9.

Now we can show that the given translation is adequate.

Theorem 6 *Let (P,T,F,M_i) be a P/T net, $M \xrightarrow{t} M'$ iff $[\![(P,T,F,M)]\!] \longrightarrow [\![(P,T,F,M')]\!]$.*

Proof. ($\Rightarrow$) Suppose $M \xrightarrow{t} M'$, so M enable the transition t, then there exists a trans-node in $[\![(P,T,F,M)]\!]$ encoding the transition t, and the corresponding place-node of ${}^\bullet t$ contain the necessary tokens to fire the transition (by translation of M), then we can apply the appropriate rule to perform the reaction reaching the configuration $[\![(P,T,F,M')]\!]$.

($\Leftarrow$) If $[\![(P,T,F,M)]\!] \longrightarrow [\![(P,T,F,M')]\!]$, there exists a matching of a rule with a sub-bigraph of $[\![(P,T,F,M)]\!]$, in particular the matched nodes have a counter part into the P/T net (P,T,F,M), so the marking M enables a transition t (corresponding to the trans-node), and then $M \xrightarrow{t} M'$. $\square$

In order to study the relation between the open Petri nets, introduced in [BCE$^+$07], and our compositional Petri nets, we need some definitions.

Definition 19 An *open Petri net* is pair $Z = (N_Z, O_Z)$, where N_Z is a P/T net and $O_Z = (O_Z^+, O_Z^-) \in 2^P \times 2^P$ are the sets of *input* and *output open places* of the net.

The notion of enabledness for transitions remains the same; besides the reactions produced by firing a transitions, we consider also the interactions with the environment, which are modelled by producing or removing a token in an open place p (denoted by $+_p$ and $-_p$ respectively).
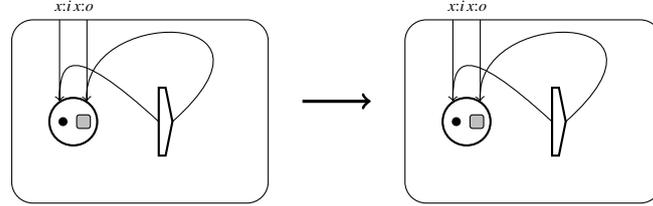
Figure 9: Example of reaction rule in the case of 1 place used as input and output.

The definition of the pre-multiset and post-multiset for the new events is the obvious one: ${}^{\bullet}+_p = \emptyset$, $+_p^{\bullet} = \{p\}$ and ${}^{\bullet}-_p = \{p\}$, $-_p^{\bullet} = \emptyset$; moreover we can lift the *firing* of a transition to the new setting: $M \xrightarrow{\delta} M'$ iff $\forall p \in P.M'(p) = M(p) - \sharp({}^{\bullet}\delta) + \sharp(\delta^{\bullet})$ (where $\delta \in T$ or $\delta = +_p, -_p$).

The encoding function for open Petri nets is analogous to the previous one (5), the only difference is replacing $\nabla_{P \times \{i,o\}}$ with $\nabla_{((P \backslash O^+) \times \{i\}) \cup ((P \backslash O^-) \times \{o\})}$. This means "opening" the places in O, which is encoded by leaving an outer downward names linked to the i or o port of a place depending if it is an input or output open place. In this way, places can be accessed by the context. With an abuse of notation, we denote with $[\![\cdot]\!]$ the new encoding function.

Now, we are ready to introduce a labelled transition system for open Petri nets.

Definition 20 The *firing LTS* associated to an open Petri net Z is the triple $\langle \bar{M}, \Lambda, \leadsto_{\bar{M},Z} \rangle$, where the states are the markings $M \in \bar{M}$, the set of label $\Lambda = \{\tau\} \cup \{+_p \mid p \in O_Z^+\} \cup \{-_p \mid p \in O_Z^-\}$ and the transition relation $\leadsto_{\bar{M},Z} \subseteq \bar{M} \times \Lambda \times \bar{M}$ includes all transition $M \xrightarrow{\lambda}_{\bar{M},Z} M'$ iff $M \xrightarrow{\lambda} M'$ and $\lambda = +_p, -_p$ or there exists $t \in T$ such that $M \xrightarrow{t} M'$ and $\lambda = \tau$.

We shall often omit the transition relation subscripts when derivable from the context.

The firing LTS has the following associated bisimulation.

Definition 21 Let Z_1, Z_2 be two open Petri nets, and $\eta : O_{Z_1} \leftrightarrow O_{Z_2}$ a correspondence between Z_1 and Z_2. A η-bisimulation between Z_1, Z_2 is a relation over markings $\mathscr{R} \subseteq \bar{M}_1 \times \bar{M}_2$ such that if $M_1 \mathscr{R} M_2$ then

- if $M_1 \xrightarrow{l} M_1'$, then there exists M_2' such that $M_2 \xrightarrow{\eta(l)} M_2'$ and $M_1' \mathscr{R} M_2'$;

- the vice versa holds;

where $\eta(+_p) = +_{\eta(p)}$, $\eta(-_p) = -_{\eta(p)}$ and $\eta(\tau) = \tau$.

Two open Petri nets Z_1, Z_2 are η-bisimilar (denoted $\sim_\eta$) if $\eta : O_1 \leftrightarrow O_2$ is a correspondence and there exists a η-bisimulation $\mathscr{R}$ such that $M_{Z_1} \mathscr{R} M_{Z_2}$, where M_{Z_1}, M_{Z_2} are the initial markings. Z_1, Z_2 are bisimilar (written $\sim$) if they are η-bisimilar for some η.

In the rest of the section, when we compare η-bisimilarity and IPO bisimilarity, we consider η defined implicitly as follows. In order to study if two bigraphical agent are bisimilar, they must have the same outer interface; hence, the correspondence of places is defined by the downward
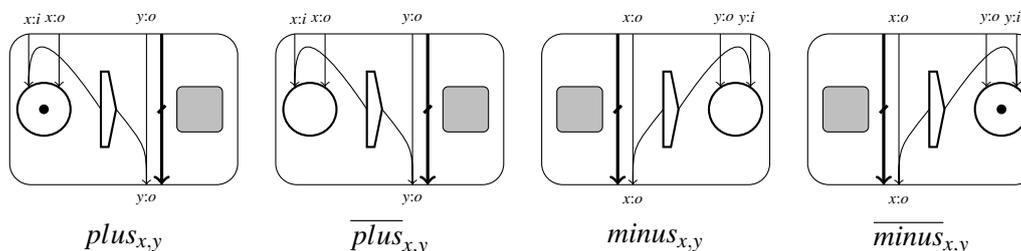
Figure 10: The labels representing the consuming or adding of tokens inside an open place.
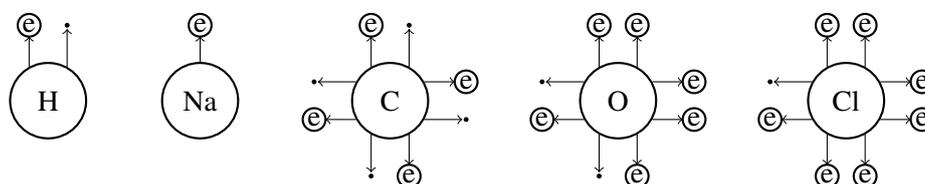
Figure 11: Example of atom encodings in directed link graphs.

names in the outer interface. In other words, an open place p_1 of Z_1 is in correspondence with an open place p_2 of Z_2 iff in their encodings $[\![Z_1]\!]$ and $[\![Z_2]\!]$, p_1 and p_2 are linked to the same names.

In order to prove the soundness result, we need to identify the IPO labels representing the labels $+_p$ and $-_p$ of the firing LTS. These labels are listed in Figure 10, where $\updownarrow$ denotes the identity over the downward names of the agent filling the context. The following results hold.

Lemma 1 *Given two open Petri nets Z_1, Z_2, and let $C[\cdot] \in \{\overline{minus}_{x,y} \otimes id_{[\![Z_1]\!]}, \overline{plus}_{x,y} \otimes id_{[\![Z_1]\!]}\}$ (where $id_{[\![Z_1]\!]} = id_{[\![Z_2]\!]}$), then $C[[\![Z_1]\!]] \sim_{IPO} C[[\![Z_2]\!]]$ iff $[\![Z_1]\!] \sim_{IPO} [\![Z_2]\!]$.*

Theorem 7 *The IPO bisimilarity is sound with respect to the firing bisimilarity.*

Proof. We have to show that $\sim_{IPO}$ is a η-bisimulation. Suppose $[\![Z_1]\!] \sim_{IPO} [\![Z_2]\!]$, and let $M_{Z_1} \overset{\lambda}{\rightsquigarrow} M'_{Z_1}$. We have to find a M'_{Z_2} such that $M_{Z_2} \overset{\lambda}{\rightsquigarrow} M'_{Z_2}$ and $[\![Z'_1]\!] \sim_{IPO} [\![Z'_2]\!]$, where Z'_1 and Z'_2 are the encodings of Z_1 and Z_2 with the new markings. This can be proved by cases on λ. $\square$

6.3 Chemical Reactions

A chemical reaction is a process describing the conversions of a chemical compositions. The chemical changes caused by a reaction always involve the motion of electrons in the forming or breaking of *chemical bonds*. For example, the *octet rule* says that atoms tend to gain, lose or share electrons so as to have eight electrons in their outer electron shell.

In this section, we give an encoding of atoms into directed link graphs, as shown in Figure 11, inspired by the well-known *Lewis structures*. We describe the atoms as nodes, and those nodes have a number of positive ports equal to the number of valence electrons. Each port is linked to an electron, represented as a node having a negative port (accepting incoming connections; for sake of simplicity we identify the node representing the electron with its port, that is, we
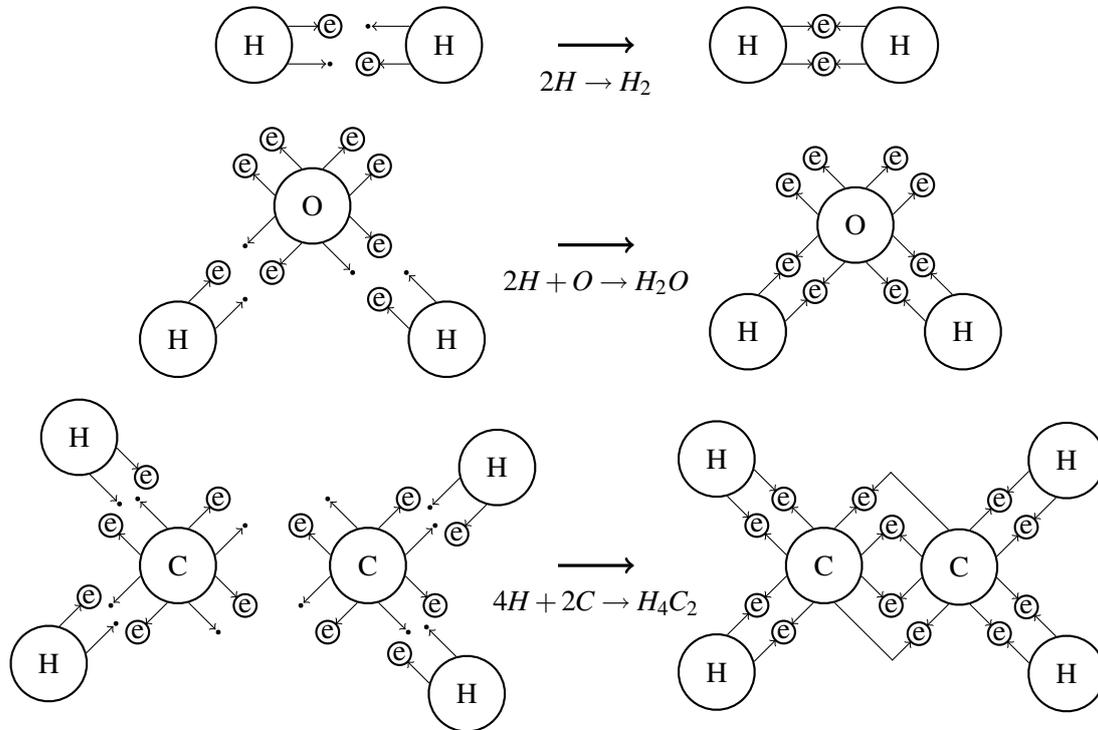
Figure 12: Example of covalent bonds among atoms.

do not force all incoming connections to be linked to a precise point of the node). Moreover, some nodes can have extra ports, that are initially linked to edges, hydrogen and oxygen can be two examples, the idea is that such a configuration describes the aim of the atom to "capture" electrons to complete its external shell; e.g. an oxygen atom has two missing electrons, so it tries to share these two electrons with a pair of hydrogen atoms forming the water molecule.

We apply this model describing the forming and breaking of bonds among atoms, here we deal with *strong bonds*, that is *covalent* and *ionic bonds*.

Some examples of covalent bonds are in Figure 12. The first shows how two hydrogen atoms can share their electrons. The second one is well-know and describes how, in the generation of a water molecule, the oxygen shares one electron with each hydrogen; in this way it gets the two missing electrons in its external orbit, conversely each hydrogen atom completes its orbit sharing an electron with the oxygen. The latter describes a more complicate situation, where two carbon atoms (each needing four electron to complete its orbit) share two electron with the other carbon atom, and the total remaining four missing electrons are provided by two pairs of hydrogen.

In Figure 13, we show an example of ionic bond: given an atom of sodium and a chlorine one, it may happen (by octet rule) that the external electron of sodium is lost by the atom and "captured" by the chlorine, forming a sodium (positive) ion and a chlorine (negative) ion. These two ions attract each other by the electrostatic force caused by the electron exchange. Finally the ions can be composed to form sodium-chloride molecule, that is the common salt.

An interesting future work is representing *weak bonds*: *hydrogen* and *van der Waals bonds*.

$$Na + Cl \rightarrow Na^+ + Cl^- \rightarrow NaCl$$
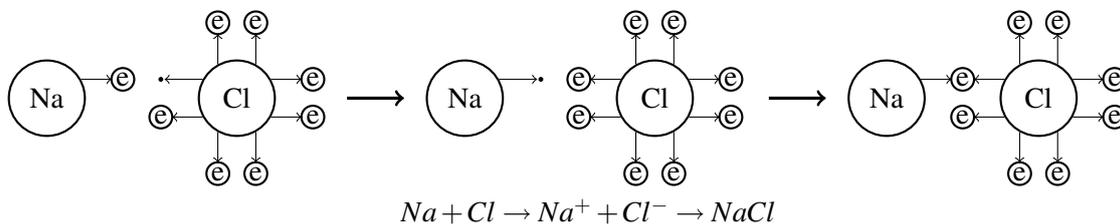
Figure 13: Examples of ion bonds among atoms.

7 Conclusions

In this paper, we have considered directed bigraphs over *polarized signatures*, a bigraphical model for concurrent, distributed system with resources and controls. The main difference with previous versions of bigraphs is the capability of nodes (i.e., systems) to ask for resource access (via the "positive ports") and to control other's requests, providing access to own resources (via the negative ports). These bigraphs have RPO and IPO constructions, thus allowing to derive systematically labelled transition systems from reactive systems, as in [JM03, GM07b], such that the corresponding bisimilarities are always congruences. These directed bigraphs admit also a notion of normal form, and a complete axiomatization.

We have exhibited the expressive power of this framework, by applying it some interesting cases: a three-tier interaction between client, server and back-end service through a firewall, the Petri nets (for which the induced IPO bisimilarity is proved to be sound with respect to η-bisimilarity of open Petri nets), and chemical reactions. All these cases are faithfully encoded as directed bigraphs with polarized signatures (possibly with sorting).

An interesting future work is to develop properly the treatment of web service interactions, extending the ideas shown in Section 6.1. In particular, we would like to give a bigraphical semantics of some formal calculus for web services, such as SCC or CC-Pi [BBC+06, BM07].

Another future development is to use this kind of bigraphs as a general framework for systems biology. Some preliminary experiment about the representation of biochemical reactions, not shown in this paper due to lack of space, are promising: ions, electrons, chemical links can be represented as controls and arcs, and the place structure can be fruitfully used to represent nesting of chemical compounds. It would be interesting to encode in directed bigraphs some important formalism for systems biology, such as the κ-calculus [DL04]. Along this line, also the possibility of adding quantitative aspects (i.e., reaction rates) sounds very promising.

Bibliography

[BBC+06] M. Boreale, R. Bruni, L. Caires, R. De Nicola, I. Lanese, M. Loreti, F. Martins, U. Montanari, A. Ravara, D. Sangiorgi, V. Vasconcelos, G. Zavattaro. SCC: A Service Centered Calculus. In *Proc. WS-FM*. LNCS 4184, pp. 38–57. Springer, 2006.

[BCE+07] P. Baldan, A. Corradini, H. Ehrig, R. Heckel, B. König. Bisimilarity and Behaviour-Preserving Reconfigurations of Open Petri Nets. In Mossakowski et al. (eds.),

Proc. CALCO. Lecture Notes in Computer Science 4624, pp. 126–142. 2007.

[BDE⁺06]   L. Birkedal, S. Debois, E. Elsborg, T. Hildebrandt, H. Niss. Bigraphical Models of Context-Aware Systems. In *Proc. FoSSaCS*. Lecture Notes in Computer Science 3921, pp. 187–201. Springer, 2006.

[BH06]   M. Bundgaard, T. T. Hildebrandt. Bigraphical Semantics of Higher-Order Mobile Embedded Resources with Local Names. *Electr. Notes Theor. Comput. Sci.* 154(2):7–29, 2006.

[BM07]   M. G. Buscemi, U. Montanari. CC-Pi: A Constraint-Based Language for Specifying Service Level Agreements. In *Proc. ESOP*. Lecture Notes in Computer Science 4421, pp. 18–32. Springer, 2007.

[BS06]   M. Bundgaard, V. Sassone. Typed polyadic pi-calculus in bigraphs. In Bossi and Maher (eds.), *Proc. PPDP*. Pp. 1–12. ACM, 2006.

[DL04]   V. Danos, C. Laneve. Formal molecular biology. *Theor. Compu. Sci.* 325, 2004.

[GM07a]   D. Grohmann, M. Miculan. Directed bigraphs. In *Proc. XXIII MFPS*. Electronic Notes in Theoretical Computer Science 173, pp. 121–137. Elsevier, 2007.

[GM07b]   D. Grohmann, M. Miculan. Reactive Systems over Directed Bigraphs. In *Proc. CONCUR 2007*. Lecture Notes in Computer Science 4703, pp. 380–394. Springer, 2007.

[GM08]   D. Grohmann, M. Miculan. An Algebra for Directed Bigraphs. In Mackie and Plump (eds.), *Proceedings of TERMGRAPH 2007*. Electronic Notes in Theoretical Computer Science 203(1), pp. 49–63. Elsevier, 2008.

[Jen08]   O. H. Jensen. *Mobile Processes in Bigraphs*. PhD thesis, University of Aalborg, 2008. To appear.

[JM03]   O. H. Jensen, R. Milner. Bigraphs and transitions. In *Proc. POPL*. Pp. 38–49. 2003.

[JM04]   O. H. Jensen, R. Milner. Bigraphs and mobile processes (revised). Technical report UCAM-CL-TR-580, Computer Laboratory, University of Cambridge, 2004.

[LM06]   J. J. Leifer, R. Milner. Transition systems, link graphs and Petri nets. *Mathematical Structures in Computer Science* 16(6):989–1047, 2006.

[Mil01]   R. Milner. Bigraphical Reactive Systems. In Larsen and Nielsen (eds.), *Proc. 12th CONCUR*. Lecture Notes in Computer Science 2154, pp. 16–35. Springer, 2001.

[Mil06]   R. Milner. Pure bigraphs: Structure and dynamics. *Information and Computation* 204(1):60–122, 2006.

[Mil07]   R. Milner. Local Bigraphs and Confluence: Two Conjectures. In *Proc. EXPRESS 2006*. Electronic Notes in Theoretical Computer Science 175(3), pp. 65–73. 2007.

[PV98]   J. Parrow, B. Victor. The Fusion Calculus: Expressiveness and Symmetry in Mobile Processes. In *Proceedings of LICS '98*. Pp. 176–185. 1998.